

# **MARKSCHEME**

**May 2006**

**MATHEMATICAL STUDIES**

**Standard Level**

**Paper 2**

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**Paper 2 Markscheme  
Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

## 1 Abbreviations

The markscheme may make use of the following abbreviations:

- M* Marks awarded for **Method**
- A* Marks awarded for an **Answer** or for **Accuracy**
- G* Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.
- R* Marks awarded for clear **Reasoning**
- AG* **Answer Given** in the question and consequently, marks not awarded.
- ft* Marks that can be awarded as **follow through** from previous results in the question

In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**. Marks to be awarded for unsupported answers are designated **G** in the mark scheme as such answers will usually arise from working performed on a graphic display calculator.

## 2 Method of Marking

- (a) All marking must be done using a **red** pen.
- (b) Marks must be noted on candidates' scripts as in the markscheme:
- Show the breakdown of individual marks using the **abbreviations** (*M1*), (*A2*) etc;
  - Write down each part mark total, as indicated on the **question paper**. These totals should be written in the **margins** of the candidates' answer booklets;
  - Write down and circle the total for each question at the end of the question.
  - Transfer the total for **each question** to the front cover sheet and write down the total mark for the paper
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.

- (e) If correct working results in a correct answer but then further working is developed, full marks are **not** always awarded. In most such cases it will be a single final answer mark that is lost. Full marks **can** be awarded if the candidate demonstrates clear understanding of the task and the result. If in doubt, consult your team leader.
- (f) Candidate drawn graphs will have a single **(A1)** available for scales and labels combined. This can be awarded if all these are present and correct, even if no graph is drawn, however, the mark should not be awarded if the scale shown is inappropriate to, or inadequate for, the required missing graph. In papers which have two candidate drawn graphs, consistent errors in showing labels or scales can follow through on the second graph, though not if the error is complete omission of these features.

**Please note:** Assignment of marks to the answers in all the following examples is for demonstration purposes only. Marks for actual examination questions will not necessarily follow the same pattern.

Question: Using Pythagoras to find a side of a triangle:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ <b>(M1)(A1)</b> (3.61 3s.f.) <b>OR</b> Answer only <b>(G2)</b>	Case (i) $\sqrt{13}$ or 3.61 or both	<b>(G2)</b>
	Case (ii) $\sqrt{9+4} = \sqrt{13}$ $= 6.50$	<b>(M1)</b> <b>(A0)</b>

Question: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)(A1)</b> <b>OR</b> Answer only <b>(G1)</b>	(i) -6/5	<b>(G1)</b>
	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ Gradient is -6/5 $y = -6x/5 + 9$	<b>(M1)</b> <b>(A1)</b> <i>(There is clear understanding of the gradient.)</i>
	(iii) $\frac{9-3}{0-5} = -\frac{6}{5}$ $y = -6x/5 + 9$	<b>(M1)</b> <b>(A0)</b> <i>(There is confusion about what is required.)</i>

Question: sine rule used to find angle  $A$ , with angle  $B$  and side  $b$  known but side  $a$  is first calculated using Pythagoras in an adjoining triangle.

Markscheme		Candidate's Script		Marking
$a = \sqrt{25 + 36} = \sqrt{61}$	(M1)(A1)	Case (i)	$a = \sqrt{61}$	(G2)
OR answer only	(G2)		$A = 55.9^\circ$	(A2)
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$	(M1)(A1)	Case (ii)	$A = 55.9^\circ$ (with no mention of $a$ )	(A2)
$A = 55.9^\circ$	(A1)			
OR answer only	(A2)			

### 3 Follow-through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with 'ft' appended to the eligible mark(s).

- If an answer resulting from follow through is extremely unrealistic (e.g. negative distances or wrong by a large order of magnitude) then the final  $A$  mark should not be awarded. If in doubt, contact your team leader.
- If a question is transformed by an error into a **different, much simpler question** then follow through might not apply or might be reduced. In this situation consult your team leader and record the decision on the candidate's script.
- To award follow through marks for a question part, **there must be working present for that part** and not just an answer based on the follow through. An isolated follow through answer, with no working, must be regarded as incorrect and receives no marks **even if it seems approximately correct**.
- Inadvertent use of radians will be penalised the first time it occurs. Subsequent use, even in later questions will normally be allowed follow through marks unless the answer is unrealistic. Cases of this kind will be addressed on an individual basis.

**Implementation:** The following examples illustrate correct use of the **follow through** process in straightforward situations.

Question: An investment problem with two different rates of interest and a total amount of \$600 split across the rates in consecutive periods:

Markscheme	Candidate's Script	Marking
(a) $\$ 600 \times 1.02$ $= \$ 612$ <b>OR</b> answer only	Case (i) (a) Final amount after 1 <sup>st</sup> period $= \$ 600 \times 1.02$ $= \$ 602$	(M1) (A1) (G2) (A0)
(b) $\$ \left( \frac{612}{2} \times 1.02 \right) + \left( \frac{612}{2} \times 1.04 \right)$ $= \$ 630.36$ <b>OR</b> answer only <i>Note: The (M1) is for splitting the value from (a) and forming a sum of products.</i>	(b) Amount after 2 <sup>nd</sup> period $= 301 \times 1.02 + 301 \times 1.04$ $= \$ 620.06$ but note Case (ii) an (M0) almost always prohibits the <b>associated (ft)</b> so	(M1) (A1)(ft) (M1)(A0) (M0)(A0)(ft)
Here the (ft) indicates a possible follow through from part (a).	(a) $\$ 600 \times 1.02 = \$ 602$ (b) $\$ 602 \times 1.04 = \$ 626.08$ Case (iii) (a) $\$ 600 \times 1.02 = \$ 602$ (b) No working. 620.06 given as answer. Case (iv) (a) \$ 612 (b) \$ 630.36	(M1)(A0) (M0)(A0)(ft) (M1)(A0) (G0)(ft) (G2) (G1)

Question: Using trigonometry to calculate angles and sides of triangles.

Markscheme		Candidate's Script	Marking
(a)	$\frac{\sin A}{3} = \frac{\sin 30}{4}$ (M1)(A1) $A = 22.0^\circ$ (A1)	(a) $\frac{\sin A}{4} = \frac{\sin 30}{3}$ $A = 41.8^\circ$	(M1) (A0) (use of sine rule but with wrong values)  (A0)  (Note: the 2 <sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded, because there was an earlier error in the same question part.)
OR	answer only (A2)		
(b)	$x = 7 \tan A$ (M1) $= 2.83$ (A1)(ft)	(b) case (i) - $x = 7 \tan A$ $= 6.26$	(M1) (A1)(ft)
OR	answer 2.83 only (G1)	but case (ii) 6.26	(G0)

#### 4 Using the Markscheme

This markscheme presents a particular way in which each question might be worked and how it should be marked.

(a) As *A* marks are normally **dependent** on the preceding *M* mark being awarded, it is **not** possible to award (M0)(A1). Once an (M0) has been awarded, all subsequent *A* marks are lost in that part of the question, even if calculations are performed correctly, until the next *M* mark, unless otherwise instructed in the markscheme. (See the finance example above).

Similarly (A1)(R0) cannot be awarded for an answer which is accidentally correct for the wrong reasons given.

**Example:** Question: (a)  $\chi^2$  calculated followed by (b) degrees of freedom found and (c) and (d) comparison to critical value. (Dependence of *A* and *R* marks.)

Markscheme		Candidate's Script	Marking
(a)	$\chi_{calc}^2 = 3.92$ (A1)	Case (i)	
(b)	$n = 4$ (A1)	(a) $\chi_{calc}^2 = 3.92$	(A1)
(c)	$\chi_{crit}^2 = 9.488$ (A1)(ft)	(b) $n = 4$	(A1)
(d)	Do not reject null hypothesis (A1)(ft) because $\chi_{calc}^2 < \chi_{crit}^2$ (R1)(ft)	(c) Don't know?	(A0)
		(d) Do not reject null hypothesis because $\chi_{calc}^2 > 0$	(A0)(ft) (R0)(ft) (A0) was awarded here because the reason is wrong.)

	Case (ii)	
	(a) $\chi_{calc}^2 = 3.92$	(A1)
	(b) $n = 4$	(A1)
	(c) $\chi_{crit}^2 = 4.488$	(A0)
	(d) Do not reject null hypothesis because $\chi_{calc}^2 < \chi_{crit}^2$	(A1)(ft) (R1)(ft)
	Case (iii)	
	(a) $\chi_{calc}^2 = 3.92$	(A1)
	(b) $n = 1$	(A0)
	(c) $\chi_{crit}^2 = 3.841$	(A1)(ft)
	(d) Reject null hypothesis because $\chi_{calc}^2 > \chi_{crit}^2$	(A1)(ft) (R1)(ft)

- (b) **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner that is consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by ‘**OR**’ *etc.* This includes alternatives obtained with a graphic display calculator. In such cases, alternative **G** mark assignments for answer only will not be repeated if this is redundant.

**Example:** Question to find the coordinates of a vertex of a given quadratic.

Working	Marks
$f(x) = 2x^2 + 7x - 3$	
$x = -\frac{b}{2a} = -\frac{7}{4}$	(M1)(A1) or (G2)
(M1) for use of $-b/2a$ , (A1) for correct answer	
$f\left(-\frac{7}{4}\right) = -\frac{146}{16} = -\frac{73}{8}$	(M1)(A1)(ft)
(M1) for using $f(-7/4)$ , (A1) for answer.	or (G1)
Coordinates are $(-7/4, -73/8)$	(A1)(ft)

<p><b>OR</b></p> <p><math>(-7/4, -73/8)</math> (with no working at all)</p> <p><b>OR</b></p> <p><math>f'(x) = 4x + 7, \quad 4x + 7 = 0</math></p> <p>so <math>x = -7/4</math>  <i>(M1) for attempting to take a derivative and setting it to 0</i>  <i>(A1) for answer</i></p> <p><math>f(-7/4) = -\frac{146}{16} = -\frac{73}{8}</math>  <i>(M1) for using <math>f(-7/4)</math>, (A1) for answer.</i></p> <p>Coordinates are <math>(-7/4, -73/8)</math></p>	<p><b>OR</b></p> <p><i>(G2)(G1)</i></p> <p><b>OR</b></p> <p><i>(M1)</i></p> <p><i>(A1)</i></p> <p><i>(M1)(A1)(ft)</i></p> <p><i>(A1)(ft)</i></p>
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(c) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
 On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

(d) As this is an international examination, all valid **alternative forms of notation** should be accepted.

Some examples of these are:

Decimal points: 1.7; 1'7; 1·7 ; 1,7.

Different descriptions of an interval:  $3 < x < 5$ ; (3, 5); ] 3, 5 [ .

Different forms of notation for set properties (e.g. complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A$

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .  
 $p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$ .

(e) Discretionary (**d**) marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, (**d**) should be used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

## 5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to 3 significant figures.

A penalty known as an **ACCURACY PENALTY (AP)** is applied if an answer is either

- (i) rounded incorrectly to 3 significant figures or
- (ii) rounded correctly or incorrectly to some other level of accuracy.

This penalty is applied to the **final answer** of a question part only. It applies **also** when an exact answer is incorrectly rounded.

**THE ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER!** Subsequent accuracy errors can be **ignored** and full marks awarded if all else is correct.

An accuracy penalty must be recorded in proximity to the incorrect answer as **(A0)(AP)**.

Examiners must record the occurrence of an accuracy penalty by writing **(AP)** next to the relevant question total on the front of the cover sheet.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. This is **NOT** an accuracy penalty. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. If the rounding is incorrect, an accuracy penalty should be applied as detailed above. Exact answers such as  $\frac{1}{4}$  can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of  $\pi$  and answers taking the form of square roots of integers (**even if exact squares**) or any rational power of an integer (*e.g.*  $\sqrt{13}$ ,  $2^{2/3}$ ,  $\sqrt[4]{5}$ ,  $\sqrt{9}$ ) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

Answers **with no supporting working** which are written correct to more than 3 significant figures should be marked according to the scheme for correct answers with no working, but with an **(AP)** then applied. When this happens, **(A2)** or **(G2)** can be split if necessary (*e.g.* **(A1)(A0)(AP)** or **(G1)(G0)(AP)**). **Unsupported** answers with less than 3 significant figures must be deemed incorrect even if they seem approximately correct.

An accuracy penalty should not be applied to an answer that is already incorrect for some other reason.

### Special cases

Answers involving units of currency can be accepted correct to 3 significant figures or correct to the nearest currency unit (*e.g.* dollar) or correct to the nearest hundredth unit (*e.g.* cent). Allow all these cases to follow through to later question parts.

An answer taken directly from the IB chi-squared statistical table can be given and used to the same level of accuracy as appears in the table (3 decimal places) or correct to 3 significant figures.

For judging equivalence between 3 significant figures and use of minutes and seconds for angles, guidelines have been issued to paper setters. This problem will be dealt with on an individual basis as the need arises.

**Examples:** The Pythagoras example used before:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (M1)(A1) (3.61 3s.f.) OR answer only (G2)	(i) 3.6 or 4	(G0)
	(ii) 3.60555	(G1)(G0)(AP)
	(iii) $\sqrt{9+4} = \sqrt{13}$ = 3.6	(M1) (A0)(AP)
	(iv) $\sqrt{9+4} = \sqrt{13}$ = 3.60555	(M1) (A0)(AP)
	(v) $\sqrt{9+4} = \sqrt{13} = 3.60$	(M1)(A0)(AP)
	(vi) $\sqrt{9+4} = \sqrt{14} = 3.74$	(M1)(A0)

If the question specified e.g. correct to 4 decimal places for the answer, then there would be one extra mark available as follows:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (M1)(A1) OR answer only (G2) (Note: requires more than 4 d.p.) = 3.6056 (4 d.p.) (A1)(ft) OR answer only (G2) OR answer 3.606 or 3.61 only (G1)	(i) 3.605551 = 3.6056 (4 d.p.)	(G2)(A1)
	(ii) $\sqrt{9+4} = \sqrt{13}$ = 3.606	(M1)(A1) (A0)
	(iii) 3.60555	(G2)(A0)
	(iv) 3.6056	(G2)
	(v) $\sqrt{9+4} = \sqrt{14}$ = 3.7417	(M1)(A0) (A1)(ft)
	(vi) $\sqrt{9-4} = \sqrt{5}$	(M0)(A0) (A1)(ft)

	= 2.2361	(Note: this is a special case, where the initial <b>(M0)</b> does not determine the final <b>(A0)</b> because the correction to <b>4dp</b> is an entirely new task.)
	(vii) 3.606	<b>(G1)</b>

**Premature Rounding**

Accuracy errors in a final answer, which result from premature rounding earlier in the same question part, should not receive an accuracy penalty. There are two situations. If there is a mark available for a prematurely rounded answer and the rounding occurs at this stage, then the inappropriate rounding should be penalised with **(A0)** but the answer can then be allowed to follow through to the end of the question. If the first stage of the answer is correct but rounded further on, then it should be penalised at an appropriate place close to where it is rounded. Some discretion should be used to deny a **(ft)** mark if the rounding is very bad and the answer far from its required value.

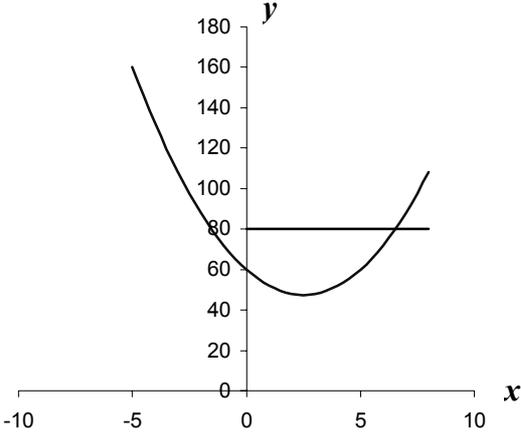
**Example:** Question: sine rule used to find angle *A*, with angle *B* and side *b* known but side *a* is first calculated using Pythagoras in an adjoining triangle.

Markscheme	Candidate's Script	Marking
$a = \sqrt{25 + 36} = \sqrt{61}$ <b>(M1)(A1)</b> <b>OR</b> answer only <b>(G2)</b>	(i) $a = \sqrt{25 + 36} = \sqrt{61}$ $= 7.8$	<b>(M1)</b> <b>(A0)</b>
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$ <b>(M1)(A1)(ft)</b>	$\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$	<b>(M1)(A1)(ft)</b>
$A = 55.9^\circ$ <b>(A1)(ft)</b> <b>OR</b> answer only <b>(G2)</b>	$A = 55.8^\circ$	<b>(A1)(ft)</b>
	(ii) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.8^\circ$	<b>(M1)(A1)</b> <b>(M1)(A0)</b> <b>(A1)(ft)</b>
	(iii) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.9^\circ$	<b>(M1)(A1)</b> <b>(M1)(A0)</b> <b>(A0)(AP)(ft)</b> <i>(even though this is the answer to the question, the rounded answer does not follow from the given</i>

	<p>(iv) <math>a = \sqrt{25 + 36} = \sqrt{61}</math></p> <p><math>\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}</math></p> <p><math>A = \sin^{-1}(0.83) = 56.1^\circ</math></p>	<p><i>working.</i></p> <p><b>(M1)(A1)</b></p> <p><b>(M1)(A0)</b></p> <p><b>(A0)</b></p>
	<p>(v) <math>a = \sqrt{25 + 36} = \sqrt{61} = 8</math></p> <p><math>\frac{\sin(A)}{8} = \frac{\sin(32)}{5}</math></p> <p><math>A = 58.0^\circ</math></p>	<p><b>(M1)(A0)</b></p> <p><b>(M1)(A1)(ft)</b></p> <p><b>(A0)(ft)</b>  <i>(The rounding is severe and the answer quite far from correct).</i></p>
	<p>(vi) <math>a = 7.8</math></p> <p><math>A = 55.8^\circ</math></p>	<p><b>(G0)</b></p> <p><b>(G0)(ft)</b>  <i>(there is no working to justify the follow through.)</i></p>

**6 Graphic Display Calculators**

Candidates will often be obtaining solutions directly from their calculators. In presenting their working they must use mathematical notation, not calculator notation. Correct answers supported only by calculator notation, without further explanation should be marked as answers without working and G marks awarded according to the mark scheme instructions.

<p><b>Q1</b></p>	<p>(a) (i) <math>a = 60</math> <math>b = 48</math></p> <p>(ii)</p>  <p>Labels and scale all points correct <i>At most one error (A1)(A0)</i> Smooth curve (not straight lines) drawn in given domain only</p> <p><i>Graphs not drawn on graph paper must be drawn very accurately to receive marks</i></p> <p>(iii) <math>x = \frac{-b}{2a} = \frac{-(-10)}{2(2)} = 2.5</math> <i>(A1) is for correct substitution</i></p> <p><b>OR</b></p> $\frac{dy}{dx} = 4x - 10$ $4x - 10 = 0, \quad x = 2.5$ <p><i>(M1) is for setting derivative to 0.</i></p> <p><b>OR</b> For correct symmetry argument leading to <math>x = 2.5</math></p> $f(2.5) = 2(2.5)^2 - 10(2.5) + 60 = 47.5$ <p><i>If graph is used and lines are drawn to the vertex from <math>x = 2.5</math> and <math>y = 47.5</math>, award (M1)(A0)(M1). If vertex is indicated on graph, with or without coordinates, Award (M1) only. If GDC is used and a sketch showing the vertex is given, award (G1) only.</i></p>	<p>(A1) (A1)</p> <p>(A1) (A2)(ft) (A1)(ft)</p> <p>(M1)(A1) (AG)</p> <p><b>OR</b></p> <p>(A1) (M1) (AG)</p> <p><b>OR</b> (R2) (AG)</p> <p>(M1) (AG)</p>	<p><i>[2 marks]</i></p> <p><i>[4 marks]</i></p> <p><i>[3 marks]</i></p>
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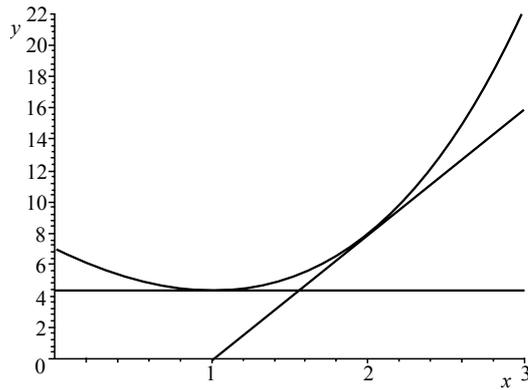
<p><b>Q2</b></p>	<p><i>Lists without set brackets are acceptable in parts (a) and (b).</i></p> <p>(i) (a) <math>A = \{1, 3, 7, 21\}</math> Award <b>(A1)</b> for any three correct with no extra members.</p> <p>(b) (i) <math>B = \{7, 14, 21\}</math> <math>A \cup B = \{1, 3, 7, 14, 21\}</math> Award <b>(A2)</b> for correct answer alone.</p> <p>(ii) <math>C'</math> is the set of all even numbers in <math>U</math> (list is okay). <math>C' \cap B = \{14\}</math></p> <p>(c) <math>A \cap B \cap C = \{7, 21\}</math> Probability is <math>\frac{2}{4}</math> (<math>\frac{1}{2}</math> or 0.5)</p> <p>(ii) (a) <math>AH = \sqrt{8^2 + 8^2} = \sqrt{128}</math> (or 11.3 or <math>8\sqrt{2}</math>) For use of Pythagoras theorem then for correct answer.</p>	<p><b>(A2)</b></p> <p><b>(A1)</b> <b>(A1)(ft)</b></p> <p><b>(A1)</b> <b>(A1)(ft)(G2)</b></p> <p><b>(A1)(ft)</b> <b>(A1)(ft)(G2)</b></p> <p><b>(M1)</b> <b>(A1)(G2)</b></p>	<p><b>[2 marks]</b></p> <p><b>[4 marks]</b></p> <p><b>[2 marks]</b></p> <p><b>[2 marks]</b></p>
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Question 2 continued

	<p>(b) <math>\cos \angle ACH = \frac{10^2 + 10^2 - (\sqrt{128})^2}{2 \times 10 \times 10}</math></p> <p><b>OR</b></p> <p><math>128 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos \angle ACH</math>          For use of cosine rule          with correct values substituted</p> <p><math>\angle ACH = 68.9^\circ</math> (68°54')</p> <p><i>Allow 68.8° (68° 48') following through with          AH = 11.3 from part (a).</i></p> <p><b>OR</b></p> <p>Triangle ACH is isosceles. Let M be the midpoint of AH.  <math>\sin \angle ACM = \frac{4\sqrt{2}}{10}</math>, hence <math>\angle ACM = 34.45^\circ</math></p> <p>For use of sine rule in appropriate triangle          with correct values substituted</p> <p><math>\angle ACH = 2 \times 34.45 = 68.9^\circ</math> (68°54')</p> <p>(c) Height AB (or equivalent) = <math>\sqrt{10^2 - 8^2} = 6</math>.  <i>Award (A1) for 6 seen.</i>          Area = <math>2 \times (8 \times 8) + 4 \times (6 \times 8)</math>  <math>= 320 \text{ cm}^2</math>  <i>Award (M1) for sum of 2 faces and 4 faces. Final (A1) must          result in an answer of 320.</i></p> <p><i>If candidate starts with 320 and works backwards to find          height of 6, award (M1)(A1). Only award final (A1) if the          candidate comments or proves that the height of 6 is          consistent with the dimensions of the prism.</i></p> <p>(d) Volume = <math>\frac{8 \times 8 \times 6}{2}</math>  <i>Award (M1) for correct numerator and (M1) for division by 2          Follow through value for height from (c).</i>  <math>= 192 \text{ cm}^3</math></p>	<p>(M1) (A1)(ft)</p> <p>(A1)(ft)(G2)</p> <p><b>OR</b></p> <p>(M1) (A1)(ft)</p> <p>(A1)(ft)(G2)</p> <p>(A1)</p> <p>(M1)(A1) (AG)</p> <p>(M1)(M1)</p> <p>(A1)(ft)(G3)</p>	<p>[3 marks]</p> <p>[3 marks]</p> <p>[3 marks]</p> <p><b>Total [19 marks]</b></p>
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<p><b>Q3</b></p>	<p>(a) <math>g(2) = \frac{1}{8}(2)^4 + \frac{9}{4}(2)^2 - 5(2) + 7</math>  <math>= 8</math></p>	<p>(M1) (A1)(G2)</p>	<p>[2 marks]</p>
	<p>(b) <math>g'(x) = \frac{1}{2}x^3 + \frac{9}{2}x - 5</math>  <i>Award (A1) for each correct term.</i>  <i>An extra constant means that the -5 is incorrect.</i></p>	<p>(A1)(A1)(A1)</p>	<p>[3 marks]</p>
	<p>(c) (i) <math>g'(2) = \frac{1}{2}(8) + \frac{9}{2}(2) - 5 = 8</math>                  For use of their derivative function                  For substitution of 2 leading to an answer of 8 only   <i>Note: beware that <math>g(2) = 8</math> also. This receives no marks in this part.</i></p> <p>(ii) <math>y = 8x + c</math>  <i>Award (A1) for <math>8x</math> seen. Must be 8.</i>                   Use of point (2, 8)  <i>Follow through with y value from part (a)</i>   <math>y = 8x - 8</math>  <i>Answer must be an equation. Allow <math>T_1 = 8x - 8</math></i></p>	<p>(M1) (A1) (AG)</p> <p>(A1)</p> <p>(M1)</p> <p>(A1)(ft)(G3)</p>	<p>[5 marks]</p>
	<p><b>OR</b></p> <p><math>8x - y = 8(2) - 8</math>  <math>y = 8x - 8</math></p> <p>(d) <math>y = \frac{35}{8}</math>                  For <math>y =</math> Allow <math>T_2 =</math>                  For <math>\frac{35}{8}</math> (4.375)</p>	<p><b>OR</b></p> <p>(A1)(M1) (A1)(ft)(G3)</p> <p>(A1)</p> <p>(A1)</p>	<p>[2 marks]</p>

(e) (i)



*The window must clearly have used x values from 0 to 3 and y values from 0 to 22. Axis labels are not required. Some indication of scale must be present. This need not be a formal scale. e.g. tick marks or single numbers on axes are adequate.*

Correct shape

Minimum in approximately correct position

(ii) Each tangent drawn at correct point.

*Lines must be straight and must be tangents.*

(A1)

(A1)

(A1)

(A1)(ft)(A1)

[5 marks]

Total [17 marks]

<p><b>Q4</b></p>	<p>(i) (a) <math>3600\left(1 + \frac{5.4}{1200}\right)^{12n} = 7200</math>  <i>(M1) for either CI formula with substitutions. (A1) for correct substitutions.</i>  <math>12n = 154.37\dots (154.4)</math> Allow <math>n = 12.86 (12.9)</math>  <math>= 155</math> months  <i>Award final (A1) for a wrong answer rounded correctly to whole months. Note that 156 is premature rounding and receives (A0).</i>  <i>For answer of 154 with no working award (G2)</i>  <i>If 0.45% or equivalent is seen and correct answer is given with no further working, award (G4) or with answers of 154 or 154.4 (G3).</i>  <i>If 0.45% or equivalent is seen with no further working and incorrect answer award (A1).</i></p> <p>(b) (i) At end of ninth year <math>3600\left(1 + \frac{5.4}{1200}\right)^{108}</math>  <i>(M1) for substituted formula (A1) for correct substitutions.</i>  <math>= 5846.50</math> euros (5847 or 5850)  <i>If 108 seen and correct answer given with no further working, award (G3). If 108 seen with incorrect answer award (A1).</i>  <i>If annual compounds used in parts (a) and (b), follow through for answer of 5779.20 euros.</i>  <i>If Simple Interest is used in parts (a) and (b), can award final (A1) in part (a) and, at most, (M1)(A1)(ft)(A0) in part (b) for answer of 5349.60.</i></p> <p>(ii) <math>I = 5846.50 - 3600 = 2246.50</math>  <math>3600 \times i \% \times 9 = 2246.50</math>  Rate <math>i \% = 6.93 \% (0.0693)</math>  Allow 6.94% (0.0694) as follow through from 5850  Answer of 18.0% receives <i>(A0)(M1)(A1)(ft)</i>  <i>If Simple Interest has been used in part (b)(i), award (A1) only for answer of 5.4%.</i></p> <p>(ii) (a) <math>t_n = ar^{n-1} \quad t_5 = 26\,000(1.03^4)</math>  For formula or list.  Correct values substituted.  <math>= \\$ 29\,263.23 (29\,263 \text{ or } 29300)</math>  For <math>n - 1 = 5</math>, and working seen, award at most <i>(M1)(A0)(A1)(ft)</i> for answer of 30141.13.</p>	<p><i>(M1)(A1)</i></p> <p><i>(A1)(G2)</i></p> <p><i>(A1)(ft)(G3)</i></p> <p><i>(M1)(A1)</i></p> <p><i>(A1)(G2)</i></p> <p><i>(A1)(ft)</i>  <i>(M1)</i>  <i>(A1)(ft)(G3)</i></p> <p><i>(M1)</i>  <i>(A1)</i>  <i>(A1)(G2)</i></p>	<p><i>[4 marks]</i></p> <p><i>[6 marks]</i></p> <p><i>[3 marks]</i></p>
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	<p>(b) (i) <math>26\,000(1.03)^{n-1} = 24\,800(1.05)^{n-1}</math>                  For equating appropriate expressions or for comparing list or for sketch from GDC showing intersection</p> <p>Correct values appearing in candidate's method.</p> <p>So a total of 3.46 years after she starts work.  <i>Allow 3 or 4 years.</i></p> <p>(ii) <math>24\,800(1.05)^3 - 26\,000(1.03)^3</math>                  Attempt to take a sensible difference.                  = \$ 298.20</p> <p><i>If (b)(i) is wrong follow through must be consistent with their decision to round up or down in (b)(i).</i></p>	<p><b>(M1)</b></p> <p><b>(A1)(ft)(A1)(ft)</b></p> <p><b>(A1)(ft)(G3)</b></p> <p><b>(M1)</b> <b>(A1)(ft)(G2)</b></p>	<p><b>[6 marks]</b>  <b>Total [19 marks]</b></p>
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<p><b>Q5</b></p>	<p>(a) (i) <math>H_0</math> : level of stress is independent of travel time  <math>H_1</math> : level of stress is not independent of travel time                  (or reasonable equivalents)</p> <p>(ii) 12.1 5.24 14.6                  20.1 8.68 24.2                  11.8 5.08 14.2  <i>(M1)</i> for attempting to calculate expected values by                  hand e.g. <math>\frac{44 \times 32}{116} = 12.1</math> etc.</p> <table style="margin-left: 40px;"> <tr><td>12</td><td>5</td><td>15</td></tr> <tr><td>20</td><td>9</td><td>24</td></tr> <tr><td>12</td><td>5</td><td>14</td></tr> </table> <p>Nearest integers</p> <p>(iii) <math>df = (r - 1)(c - 1) = (3 - 1)(3 - 1) = 4</math></p> <p>(iv) <math>\chi^2 = 9.83</math> (1)  <b>OR</b> <math>\chi^2 = 9.277</math>..... if calculated from integer values</p> <p>(v) For <math>\chi^2 = 9.83</math> Do not accept <math>H_0</math> :                  (Level of stress is not independent of travel time or                  reasonable equivalent)                  because <math>\chi^2_{\text{calc}} &gt; \chi^2_{\text{crit}}</math> <b>or</b> <math>p\text{-value} &lt; 0.05</math></p> <p><b>OR</b>                  For <math>\chi^2 = 9.278</math> Accept <math>H_0</math> :                  because <math>\chi^2_{\text{calc}} &lt; \chi^2_{\text{crit}}</math> <b>or</b> <math>p\text{-value} &gt; 0.05</math></p> <p><i>Note: a correct reason must be given for the                  (A1) to be awarded.</i></p>	12	5	15	20	9	24	12	5	14	<p>(A1)                  (A1)(ft)</p> <p>(M1)(A1)(G2)</p> <p>(A1)(G3)</p> <p>(M1)(AG)</p> <p>(G2)</p> <p><b>OR</b> (M1)(A1)</p> <p>(A1)(ft)</p> <p>(R1)(ft)</p> <p><b>OR</b>                  (A1)(ft)                  (R1)(ft)</p>	<p>[2 marks]</p> <p>[3 marks]</p> <p>[1 mark]</p> <p>[2 marks]</p> <p>[2 marks]</p> <p>[1 mark]</p> <p>[1 mark]</p>
12	5	15										
20	9	24										
12	5	14										
<p>(b)</p>	<p>(i) <math>r = 0.667</math></p> <p>(ii) Stress rating increases as travel time increases                  (or reasonable equivalent e.g. <math>y</math> increases as <math>x</math> increases).   <i>Do not accept "positive correlation"</i></p>	<p>(A1)</p> <p>(R1)</p>	<p>[1 mark]</p> <p>[1 mark]</p>									

	<p>(iii) <math>y = 0.181x + 2.22</math>  for <math>0.181x</math> and  for <math>2.22</math>  For <math>y = 2.22x + 0.181</math>, award <b>(A0)(A1)(ft)</b></p> <p>(iv) Putting <math>x = 45</math>  <math>0.181 \times 45 + 2.22</math>  <math>= 10.365</math> (10.4)  Allow 10 or 11 only if the method is shown and is correct.   <i>Allow follow through only if method shown.</i></p> <p>(v) not reliable ...  Because result is outside the data range <b>or</b> because the correlation coefficient not high or the sample is small <b>or</b> responses are subjective.  Award <b>(R1)</b> for any of the above. A correct reason must be given to award the <b>(A1)</b>.</p>	<p><b>(A1)</b>  <b>(A1)</b></p> <p><b>(M1)</b>  <b>(A1)(ft)(G2)</b></p> <p><b>(A1)</b>  <b>(R1)</b></p>	<p><b>[2 marks]</b></p> <p><b>[2 marks]</b></p> <p><b>[2 marks]</b></p>
		<b>Total [18 marks]</b>	